An Efficient Forecasting Procedure for The MDCEV Model

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Outline

- Introduction and Motivation
  - Multiple discrete-continuous choices
  - Kuhn-Tucker (KT) demand systems and the MDCEV model
  - Forecasting with the MDCEV model
- Properties of the MDCEV Model
- An Efficient Forecasting Algorithm
- Application Results

Multiple Discrete-Continuous Choices

- Several consumer choice situations are characterized by:
  - Discrete components: "what goods/alternatives to choose"
  - Continuous components: "how much to consume"
  - Multiple Discreteness: choice of multiple alternatives that are imperfect substitutes to one another
- Examples:
  - Activity participation and time-use (Bhat 2005; Habib and Miller 2009)
  - Household vehicle type holdings and usage (Ahn et al. 2008; Bhat et al., 2009)
  - Household energy consumption (Energy type & usage choices; Pinjari and Bhat, 2010)
  - Household water consumption
  - Household expenditures (Ferdous et al., 2010)
  - Grocery purchases (Brand choice and purchase quantity; Kim et al., 2002)

Modeling Methods

- Multivariate Discrete-Continuous frameworks
  - Not based on utility maximization framework for multiple discreteness
  - Fundamental consumer behaviors (e.g., satiation effects) are not captured
- Utility maximization-based approaches
  - Direct utility-based Kuhn-Tucker (KT) Demand Systems
    - Hanemann (1979), Wales and Woodland (1983), Kim et al. (2002), Phaneuf et al. (2000), von Haefen et al. (2004), Bhat (2005, 2008)
  - Indirect utility-based dual approaches
    - Lee and Pitt (1986), Phanuef (1999)

Kuhn-Tucker (KT) Demand Systems

- Earlier KT demand systems
  - Hanemann (1979) and Wales and Woodland (1983), but not many applications.
- Recent KT demand systems
  - Kim et al. (2002)
  - von Haefen et al. (2004), von Haefen and Phaneuf (2005), and others
  - Bhat (2005, 2008) → **The MDCEV Model**
- The basic MDCEV framework is being extended in several directions
  - MDCNEV (Pinjari and Bhat, 2010)…Nested extreme value error structures
  - MDCGEV (Pinjari, 2010)…GEV error structures
  - GMDCEV (Bhat and Pinjari, 2010)…complementarity among choice alternatives
- Increasing number of KT demand model applications in the recent past

Gaps in Research

- Despite the many developments and empirical applications, forecasting and policy analysis with the MDCEV model and other KT demand systems has been very difficult
- Currently available forecasting procedures are computationally expensive and potentially inaccurate
- This has severely limited the applicability of these model systems for practical forecasting and policy analysis

Objectives of this Research

- To develop a simple, efficient, and practically feasible forecasting procedure for the MDCEV Model
- Generalize the forecasting procedure to other KT demand model systems
- Specifically, we develop a forecasting algorithm that builds on simple, yet insightful explorations with the Kuhn-Tucker conditions of optimal utility that shed new light on the properties of the MDCEV model

The MDCEV Framework

- MDCEV and other KT demand systems are based on:

  Resource allocation formulation
  - Consumers allocate a limited amount of resources (e.g., time, money) to consume goods/alternatives to maximize the utility of their consumption

  Random utility maximization (RUM)
  - A stochastic utility framework is used to recognize analyst's lack of awareness of all factors affecting consumer decisions

  Non-linear utility framework
  - To accommodate satiation and variety seeking (i.e., multiple discreteness)

A stochastic, constrained, non-linear utility optimization formulation

$$\text{Maximize } U(t_1, t_2, \ldots t_k, \ldots, t_K) \quad \text{subject to: } \sum_{k=1}^{K} t_k = T, \; t_k \geq 0$$

The MDCEV Model

Consumers are assumed to maximize the following utility function:

$$U(\boldsymbol{t}) = \frac{1}{\alpha_1}\psi_1 t_1^{\alpha_1} + \sum_{k=1}^{K} \psi_k \frac{\gamma_k}{\alpha_k}\left\{\left(\frac{t_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right\} \quad \text{st: } \sum_{k=1}^{K} t_k = T,\ t_k \geq 0\,\forall k$$

$\boldsymbol{t}$: vector of consumption quantities $\boldsymbol{t} = (t_1, t_2, \ldots t_k, \ldots t_K)$

ψ_k : baseline (marginal)utility

α_k : allows diminishing marginal utility (hence multiple discreteness)

γ_k : allows corner solutions (i.e., some goods may not be chosen)

$\psi_k = \beta' z_k + \varepsilon_k$

z_k : alternative attributes and consumer characteristics

ε_k : stochastic (or error) term for k^{th} good

- Lagrangian for constrained, non-linear utility maximization:

$$L = \frac{1}{\alpha_1}\psi_1 (t_1)^{\alpha_1} + \sum_{k=2}^{K} \psi_k \left[\frac{\gamma_k}{\alpha_k}\left(\frac{t_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right] - \lambda\left(\sum_{t=1}^{K} t_k - T\right)$$

- Kuhn-Tucker (KT) conditions:

$$\psi_1 \left(t_1^*\right)^{\alpha_1 - 1} - \lambda = 0$$

$$\psi_k \left(\frac{t_k^*}{\gamma_k}+1\right)^{\alpha_k - 1} = \lambda \quad \text{if } t_k^* > 0$$

$$\psi_k \left(\frac{t_k^*}{\gamma_k}+1\right)^{\alpha_k - 1} < \lambda \quad \text{if } t_k^* = 0$$

- These stochastic KT conditions can be used to derive consumption probabilities
- Type-1 extreme value error terms → closed-form consumption probabilities

Forecasting with the MDCEV Model

- Forecasting with the MDCEV model involves solving the stochastic, constrained, non-linear utility maximization problem
- Unfortunately, there is no analytical solution to the problem
- A combination of simulation and optimization methods is required
  - The analyst must carryout constrained non-linear optimization at each simulated value of unobserved heterogeneity (i.e., error terms) to obtain the corresponding conditional forecasts
  - The conditional consumption forecasts evaluated over the entire simulated distribution of unobserved heterogeneity can be used to derive the distributions of consumption forecasts

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- Current methods to obtain conditional forecasts

  Enumerative optimization: Enumerate all possible choice baskets. Bruteforce method; computationally burdensome even with a modest number of choice alternatives.

  Gradients-based iterative optimization: Begin with an initial solution for the consumption values, and update the solution using gradients of the utility function until a desired level of accuracy is reached.
  - Iterative → computationally intensive
  - Potential convergence (and accuracy) issues

  von-Haefen et al. (2004) method (Numerical Bisection): Builds on the insight that knowing the consumption value of one alternative leads to all other consumption values. Much efficient than generic gradients-based optimization algorithms, but still iterative in nature.
- Need an efficient non-iterative optimization procedure

Properties of the MDCEV Model

- Property (1): The baseline marginal utility of a chosen alternative is always greater than that of a not-chosen alternative

$$\psi_i > \lambda > \psi_j \quad \text{if alt '}i\text{' is chosen and alt '}j\text{' is not chosen}$$

- It naturally follows from this property that:
  - when all the alternatives available to a consumer are arranged in the descending order of their baseline marginal utility values,
  - and if it is known that the number of chosen alternatives is ***M***,
  - then one can easily identify the chosen alternatives as the first ***M*** alternatives in the arrangement.

If the number of chosen alternatives is *M*, then the upper bound and lower bound for λ are ψ_M and ψ_{M+1}, respectively.

- Property (2):

If the chosen alternatives are known (say *M* alternatives are chosen), and when all the satiation parameters (i.e., α_k parameters) are equal, the Lagrange multiplier (λ) as well as the optimal consumptions can be expressed in an analytical form.

- Analytical expression for the Lagrange multiplier λ when alphas (α_k) are equal across all alternatives:

$$\lambda = \left(\frac{E + \sum_{k=2}^{M} \gamma_k}{(\psi_1)^{\frac{1}{1-\alpha}} + \sum_{k=2}^{M} \gamma_k (\psi_k)^{\frac{1}{1-\alpha}}} \right)^{\alpha-1} \text{..............................(1)}$$

- Closed-from consumption expressions when all alphas (α_k) are equal:

$$t_1^* = \left(\frac{\lambda}{\psi_1} \right)^{\frac{1}{\alpha-1}} \text{..(2)}$$

$$t_k^* = \left(\left(\frac{\lambda}{\psi_k} \right)^{\frac{1}{\alpha-1}} - 1 \right) \gamma_k; \ \forall k = (2,3,...,M) \text{.................(3)}$$

- The only catch is that the number of chosen alternatives is unknown *apriori*.
- Thus, we build an algorithm that:
  - Begins with an assumption that only one alternative is chosen
  - Verify if the KT conditions are met
  - If the KT conditions are not met, the number of chosen alternatives is increased by one and the KT conditions are verified again
  - These steps are repeated until the KT conditions are met

An Efficient Forecasting Algorithm

Step 0: Assume that only one alternative is chosen ($M = 1$)

Step 1: Arrange all the K alternatives available to the consumer in the descending order of their baseline marginal utility (ψ_k) values

Step 2: Compute the value of Lagrange multiplier λ using Equation 1

Step 3:

If $\lambda > \psi_{M+1}$ (i.e., if the KT conditions are satisfied)

- Compute optimal consumptions of the first M alternatives using Equations 2 & 3
- Set the optimal consumptions of the remaining K-M alternatives as zero

Else: go to step 4

Step 4: $M = M$+1 (update the no. of chosen alternatives)

If $M = K$: Compute optimal consumptions using Equations 2 & 3

Else: Go to step 2

Features of the Algorithm

- Enumerates choice baskets in the most efficient fashion
  - The number of times the algorithm enumerates choice baskets is equal to the number of chosen alternatives
- Non-iterative
- Can be easily generalized for other KT demand systems (see paper)
- The only disadvantage is that the algorithm is designed for utility functions with equal alpha parameters across all alternatives
  - We have recently overcome this problem as well, albeit with an iterative algorithm (see paper)

Preliminary Experiments

- An MDCEV Model was estimated and applied to an empirical context of household transportation expenditures in the following 7 alternative categories:
  - Vehicle Purchases
  - Gasoline and Motor Oil
  - Vehicle Insurance
  - Vehicle Maintenance
  - Air Travel
  - Public Transportation
  - All other expenditures and savings (numeraire outside good)
- Data
  - Obtained from the Consumer Expenditure Survey conducted by the U.S. Bureau of Labor Statistics (BLS) (see Ferdous et al., 2010)
  - Expenditures data from 4000 households
  - Household income considered as the budget constraint

To forecast the expenditure patterns
of 4000 households
in 7 alternative categories,
with 500 sets of error term draws for each household:

Proposed Algorithm

- **Takes less than 2 minutes**
- No convergence-related problems, thanks to its non-iterative nature. More accurate than the iterative procedure

CML module in GAUSS

- **Would take 2 days**
- Possible convergence issues leading to inaccurate forecasts

In another application (Eluru et al. 2010) with:
62 choice alternatives,
2000 individual observations, and
100 sets of error term draws for each household,
the algorithm took about 10 minutes.

Another application: Residential Energy Consumptions

- Households' choice (and consumption amount) of different types of energy
- Choice alternatives:
  - Electricity
  - Natural Gas
  - Fuel Oil
  - LPG
  - All other household expenditures and savings clubbed into a numeraire good
- Budget: Annual household income
- Data:
  - 2005 Residential Energy Consumption Survey (RECS)
  - 4382 Household records
  - Model estimated using data from 2473 households
  - Forecasting was performed on all 4382 households

Forecasting computation-times (seconds) with the proposed algorithm

# households / # Error draws	1000	2000	3000	4000	4382
100	2.5	5.1	7.4	10.2	10.6
200	4.8	9.6	14.8	19.6	20.8
300	7.1	14.3	21.2	28.7	30.9
400	9.5	18.8	28.3	37.1	41.9
500	11.8	23.5	35.4	46.9	51.6

The proposed algorithm takes 10.64 seconds to predict the energy consumption patterns of 4382 households, with 100 sets of error term draws for each household.

The gradients-based iterative forecasting procedure would take 15 days to do the same.

	Predicted		Observed	
	Average annual household expenditure ($)	Average annual Household consumption (Million BTU)	Average annual household expenditure ($)	Average annual household consumption (Million BTU)
Outside good	45766		45805	
Electricity	1100	38	1116	39
Natural Gas	548	50	487	44
Fuel Oil	157	11	149	10
LPG	84	4	98	5

Predicted Changes in U.S. Residential Energy Expenditures due to Climate Change

	450° Fahrenheit Increase in Annual Cooling Degree Days	
	Predicted changes in U.S. residential energy expenditures (Billion $)	Predicted changes in energy consumption (Trillion BTU)
	Average (Standard error)	Average (Standard error)
Outside good	-5.281 (0.454)	
Electricity	5.581 (0.478)	189.78 (16.31)
Natural Gas	-0.169 (0.016)	-15.72 (1.44)
Fuel Oil	-0.074 (0.010)	-5.03 (0.70)
LPG	-0.057 (0.009)	-2.70 (0.41)

Conclusion

- Forecasting with MDCEV and other KT demand systems has been very difficult due to analytical and computational complexities
- This paper proposes a forecasting algorithm that is:
  - Simple and intuitive
  - Highly efficient
  - Non-iterative
  - More accurate compared to current iterative algorithms
- The algorithm can be easily generalized for other KT demand systems
- The algorithm can be used not only for forecasting, but also to generate datasets for conducting experiments with the MDCEV model
- This algorithm enables the use of MDCEV frameworks for practical forecasting and policy analysis

Thank you

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