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The MDCEV Model: Theory and Formulation

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Multiple-Discrete Continuous (MDC) Choices

Multiple discrete-continuous

- Choice of multiple alternatives *simultaneously* with a *continuous* component

- Examples
  - Vehicle type holdings and usage
  - Activity type choice and duration of participation
  - Airline fleet mix and usage
  - Brand choice and purchase quantity

- Capable of accommodating multiple choices

MDC Models

Karush-Kuhn-Tucker (KKT) approach

- Wales and Woodland (1983)
- Satisfies the restrictions of utility theory
- Allows to incorporate a combination of corner solutions (zero consumption) for some goods and interior solutions (strictly positive consumption) for other goods
- Random utility distribution assumptions lead to a complicated likelihood function that entails multi-dimensional integration

MDC Models

Kim et al. (2002)

- Used the GHK simulator to evaluate the multivariate normal integral appearing in the likelihood function in the KKT approach
- Used a generalized variant of the well-known translated constant elasticity of substitution (CES) direct utility function

Bhat (2005)

- Based on the generalized variant of the translated CES utility function but with a multiplicative log-extreme value error term
- Labeled as the multiple discrete-continuous extreme value (MDCEV) model
- MDCEV model collapses to the MNL

Objectives

Specify a MDC model with:

- Normal error term (not iid)
- Random coefficients (not iid)

Estimate the proposed model using MACML (Bhat, 2011)

Utility Function

$$U(\boldsymbol{x}_q) = \sum_{k=1}^{K} \gamma_k \psi_{qk} \left(\left(\frac{x_{qk}}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right)$$

x_{qk}	consumption quantity
α_k, γ_k	model parameters
ψ_{qk}	baseline marginal utility
$q = 1,2,\ldots,Q$	decision maker (individuals)
$k = 1,2,\ldots, K$	goods (alternatives)

- Role of ψ_{qk}

$$\frac{\partial U(\boldsymbol{x}_{qk})}{\partial x_{qk}} = \psi_{qk} \left(\frac{x_{qk}}{\gamma_{qk}} + 1 \right)^{\alpha_{qk}-1}$$

- ψ_{qk} : baseline (at zero consumption) marginal utility
- ψ_{qk}/ψ_{ql} : marginal rate of substitution at zero consumption
- Higher baseline implies less likelihood of a corner solution for good k

Utility function

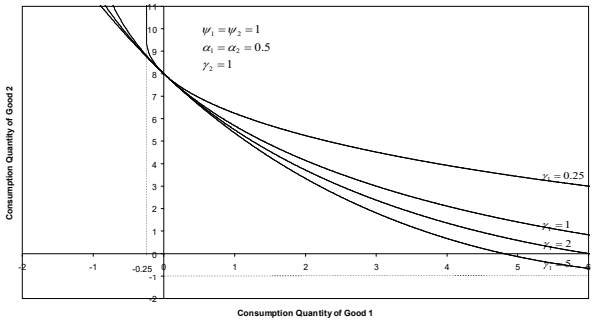
- Role of γ_{qk} $(\gamma_{qk}>0)$

$$\text{Slope } (x_{q1},x_{q2}) = \frac{\partial U(\boldsymbol{x}_q)/\partial x_{q1}}{\partial U(\boldsymbol{x}_q)/\partial x_{q2}} = \frac{\left(\frac{x_{q2}}{\gamma_{q2}} + 1 \right)^{1-\alpha_{q2}}}{\left(\frac{x_{q1}}{\gamma_{q1}} + 1 \right)^{1-\alpha_{q1}}} \times \frac{\psi(x_{q1})}{\psi(x_{q2})}$$

$$\text{At } x_{q1} = -\gamma_{q1}, \text{ slope} = \infty$$

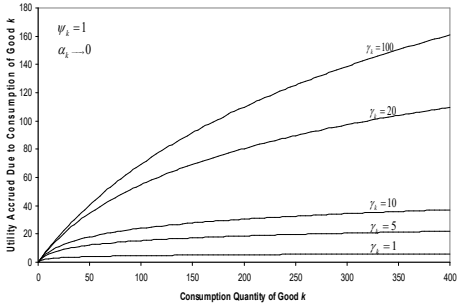
$$\text{At } x_{q2} = -\gamma_{q2}, \text{ slope} = 0$$

- Indifference Curves



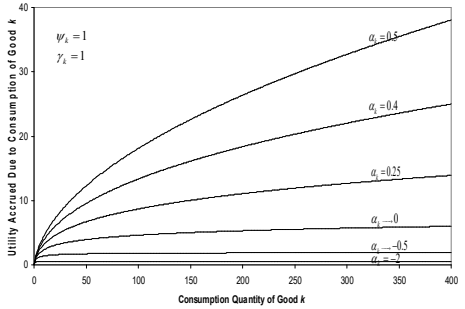
Indifference Curves Corresponding to Different Values of γ_1

- Role of $\gamma_{qk}(\gamma_{qk}>0)$



Effect of γ_{qk} Value on Good k's Subutility Function Profile

- Role of α_{qk}



Effect of α_{qk} Value on Good k's Subutility Function Profile

Utility Maximization

$$\max_{x_q} \; U(\boldsymbol{x}_q) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_{qk} \left(\left(\frac{x_{qk}}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right)$$

$$st \; \sum_{k=1}^{K} p_k x_{qk} = E_q$$

$$\psi_{qk} = \exp\left(\boldsymbol{\beta}' \boldsymbol{z}_{qk} + \xi_{qk}\right)$$

Lagrangian and KT Conditions

$$L = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \left[\exp(\boldsymbol{\beta}' \boldsymbol{z}_{qk} + \xi_{qk})\right] \left\{ \left(\frac{x_{qk}}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\} - \lambda \left[\sum_{k=1}^{K} p_k x_{qk} - E_q \right]$$

$$\left[\exp(\boldsymbol{\beta}_q' \boldsymbol{z}_{qk} + \xi_{qk})\right] \left(\frac{x_{qk}^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k = 0 \text{ , if } x_{qk}^* > 0, \; k = 1, 2, \ldots, K$$

$$\left[\exp(\boldsymbol{\beta}_q' \boldsymbol{z}_{qk} + \xi_{qk})\right] \left(\frac{x_{qk}^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k < 0 \text{ , if } x_{qk}^* = 0, \; k = 1, 2, \ldots, K$$

First Order (KKT) Conditions

$$x_{qk}^* > 0 \qquad V_{qk} + \boldsymbol{\beta' z_{qk}} + \xi_{qk} = \lambda_q$$

$$x_{qk}^* = 0 \qquad V_{qk} + \boldsymbol{\beta' z_{qk}} + \xi_{qk} < \lambda_q$$

$q = 1,2,\ldots,Q$ individuals

$k = 1,2,\ldots,K$ alternatives

where $V_{qk} = (\alpha_k - 1)\ln\left(\frac{x_{qk}^*}{\gamma_k} + 1\right) - \ln p_{qk}$

Let $y_{qk} = V_{qk} + \boldsymbol{\beta' z_{qk}} + \xi_{qk}$

$$x_{qk}^* > 0 \qquad y_{qk}^* = y_{qk} - y_{qm_q} = 0$$

$$x_{qk}^* = 0 \qquad y_{qk}^* = y_{qk} - y_{qm_q} < 0$$

m_q is the first good with non-zero consumption for consumer q

- The MDCEV model structure
  - Probability of the consumption pattern of the goods (rather than the expenditure pattern) is

$$P\left(x_1^*, x_2^*, x_3^*, \ldots, x_M^*, 0, 0, \ldots, 0\right)$$

$$= \frac{1}{p_1} \cdot \frac{1}{\sigma^{M-1}} \left[\prod_{i=1}^{M} f_i\right] \left[\sum_{i=1}^{M} \frac{p_i}{f_i}\right] \left[\frac{\prod_{i=1}^{M} e^{V_i/\sigma}}{\left(\sum_{k=1}^{K} e^{V_k/\sigma}\right)^M}\right] (M-1)!,$$

where

$$f_i = \left(\frac{1-\alpha_1}{x_i^* + \gamma_i}\right)$$

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An Application

Data Source & Dependent Variables

- Puget Sound household travel survey (April-June, 2014)
- Dependent variables
  - Residential density (hh/sq. mile) : less than 750, 750-1,999, 2,000-2,999, 3,000 or above (nominal variable)
  - Household average commute distance in miles (continuous variable)
  - Auto ownership (count variable)
  - Out-of-home discretionary activity: Personal business, shopping, recreation, dining out, social, and serve passenger. In-home is an outside good. (MDC variable)
- Unit of analysis: Household (For MDC: It is the fraction of time spent participating in different activities by the household)
- Initial sample size: 6,036 households
- Final sample size: 3,637

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Indicator variable: Ordinal variables

Attitudinal Question	Response				
	Very Unimportant 1	2	3	4	Very Important 5
How important when choosing current home:					
Having a walkable neighborhood and being near to local activities	5.5%	7.6%	11.1%	32.3%	43.5%
Being close to public transit	15.4%	12.0%	17.0%	24.8%	30.8%
Being within a 30-minute commute to work	6.6%	6.5%	10.0%	24.4%	52.5%
Quality of schools (K-12)	31.2%	7.5%	26.7%	14.6%	20.0%
Having space and separation from others	9.2%	13.7%	21.8%	34.3%	21.0%
Being close to the highway	12.7%	16.0%	21.4%	38.0%	11.9%

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Dependent variable: Count variable

Motorized Vehicle Count	Frequency						
	0	1	2	3	4	5	>6
Number	304	1,378	1,354	413	135	36	17
%	8.4	37.8	37.2	11.4	3.7	1.0	0.5

Dependent variable: MNP variable

Residential Density (households per sq. mile)	Number of observations (%)
<750	478 (13.2)
750-2,000	866 (23.8)
2,000-3,000	525 (14.4)
>3,000	1,768 (48.6)

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Dependent variable: MDC variable

Activity	Participation (%)	Mean fraction	Number of households (% of total number) spent time…	
			Only in activity type	In other activity types too
In home (IH)	3,637 (100.0)	0.780	533 (14.7)	3,104 (85.3)
Personal Business	1,607 (44.2)	0.202	216 (13.4)	1,391 (86.6)
Shopping	1,664 (45.8)	0.060	355 (21.3)	1,309 (78.7)
Recreation	1,011 (27.8)	0.131	148 (14.6)	863 (85.4)
Dining Out	1,092 (30.0)	0.081	203 (18.6)	889 (81.4)
Social	659 (18.1)	0.180	82 (12.4)	557 (87.6)
Serve Passenger	751 (20.6)	0.047	26 (3.5)	725 (96.5)

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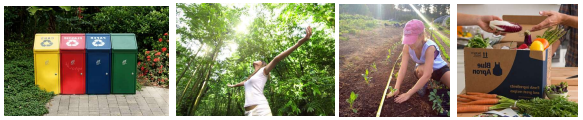
Latent Variables

❑ Green lifestyle propensity (GLP)



❑ Luxury lifestyle propensity (LLP)

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Indicators

- Green lifestyle propensity (GLP)
  - Average household commute distance (continuous)
  - Auto ownership (count)
  - Having a walkable neighborhood (ordinal with five scale)
  - Being close to public transit (ordinal with five scale)
  - Being within a 30-minute commute to work (ordinal with five scale)
- Luxury lifestyle propensity (LLP)
  - Auto ownership (count)
  - Having space and separation from others (ordinal with five scale)
  - Quality of schools (ordinal with five scale)
  - Being close to the highway (ordinal with five scale)

Latent Variable Determinants

Green Lifestyle Propensity:

- Lower income households have a higher GLP relative to higher income.
- Households with a high fraction of young adults (less than the age of 34 years) have a higher GLP relative to those with a low fraction of young adults.
- Higher GLP associated with households with a high fraction of women (relative to a low fraction of women) and a high fraction of well-educated individuals in the household (relative to a low fraction of well-educated individuals).

Luxury Lifestyle Propensity:

- LLP increases with household income
- LLP increases with the number of children in the household
- LLP increases with the age of household members

Correlation between GLP and LLP:

Negative correlation (a green lifestyle is associated with conservative consumption of resources, while a luxury lifestyle correlates with extravagant living)

Effect of Latent Constructs

- Households with a GLP disposition
  - will prefer to reside in high density neighborhoods close to their workplace,
  - own few or no vehicles,
  - and engage more in IH activities and OH social and active recreation activities
- Households with an LLP disposition
  - will be inclined to locate in low to medium density neighborhoods,
  - own many vehicles,
  - and potentially be engaged in more OH shopping and dining out activities

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Endogenous Effects

- Residential density of the household's location impacts the household auto ownership level and household activity time use.
  - Residing in lower (higher) density neighborhoods leads to a higher (lower) auto ownership level and lower (higher) baseline preferences for OH recreational activities, shopping, and dining out.
  - Time investment in serve passenger activity increases as one moves from the highest density neighborhoods to progressively lower density neighborhoods.
- Commute distance impacts only time use.
  - Households with longer commute distances spend more time on shopping, recreation, and dining out.

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Examining "True" Effects of Neo-Urbanist Densification Efforts

Table: Treatment Effects Corresponding to Transplanting a Random Household from a Lowest Density Neighborhood (<750 hh/sq. mile) to Highest Density Neighborhood (>3000 hh/sq. mile) (standard error in parenthesis)

Variable	ATE from GHDM	ATE from IHDM	% Difference Attributable to	
			"True" Effect	Self-Selection Effect
Vehicle ownership	-0.143 (0.011)	-0.340 (0.021)	42	58
Participation on				
Personal business	-0.037 (0.013)	-0.041 (0.013)	90	10
Shopping	0.011 (0.004)	0.019 (0.007)	65	35
Recreation	0.134 (0.021)	0.190 (0.014)	71	29
Dining out	0.094 (0.020)	0.119 (0.021)	79	21
Social	-0.056 (0.014)	-0.078 (0.017)	72	28
Serve Passenger	-0.156 (0.033)	-0.162 (0.025)	96	4

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Thank you!

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